GRE®

Graduate Record Examinations®

Preparing for the Verbal and Quantitative Sections of the GRE General Test

Sample Questions with Explanations

ETS Educational Testing Service

Sample Questions with Explanations

The sample questions that follow are organized by content category and represent the types of questions included in the General Test. The purpose of these questions is to provide some indication of the range of topics covered in the test as well as to provide some additional questions for practice purposes. **These questions do not represent either the length of the actual test or the proportion of actual test questions within each of the content categories.**

VERBAL ABILITY

The verbal ability measure is designed to test the ability to reason with words in solving problems. Reasoning effectively in a verbal medium depends primarily upon the ability to discern, comprehend, and analyze relationships among words or groups of words and within larger units of discourse such as sentences and written passages.

The verbal measure consists of four question types: *analogies, antonyms, sentence completions,* and *reading comprehension* sets. The examples of verbal questions in this section do not reflect precisely the difficulty range of the verbal measure.

ANALOGIES

Analogy questions test the ability to recognize the relationship that exists between the words in a word pair and to recognize when two word pairs display parallel relationships. To answer an analogy question, you must formulate the relationship between the words in the given word pair and then must identify the answer choice containing words that are related to one another in most nearly the same way. Some examples of relationships that might be found in analogy questions are relationships of kind, size, spatial contiguity, or degree.

Some approaches that may be helpful in answering analogy questions:

- Before looking at the answer choices, try to establish a precise relationship between the words in the given pair. It is usually helpful to express that relationship in a phrase or sentence. Next, look for the answer choice with the pair of words whose relationship is closest to that of the given pair and can be expressed in a similar fashion.
- Occasionally, more than one of the answer choices may seem at first to express a relationship similar to that of the given pair. Try to state the relationship more precisely or identify some aspect of the relationship between the given pair of words that is paralleled in only *one* choice pair.
- Remember that a single word can have several different meanings. Check to be sure you have not overlooked a possible second meaning for one of the words.
- *Never* decide on the best answer without reading *all* the answer choices.
- Practice recognizing and formulating relationships between word pairs. You can do this with the following sample questions.

Directions: **In each of the following questions, a related pair of words or phrases is followed by five lettered pairs of words or phrases. Select the lettered pair that best expresses a relationship similar to that expressed in the original pair.**

1. COLOR : SPECTRUM : : (A) tone : scale (B) sound : waves (C) verse : poem (D) dimension : space (E) cell : organism

The relationship between *color* and *spectrum* is not merely that of part to whole, in which case (E) or even (C) might be defended as correct. A *spectrum* is made up of a progressive, graduated series of *colors*, as a *scale* is of a progressive, graduated sequence of *tones*. Thus, (A) is the correct answer choice. In this instance, the best answer must be selected from a group of fairly close choices.

2. HEADLONG : FORETHOUGHT : : (A) barefaced : shame (B) mealymouthed : talent (C) heartbroken : emotion (D) levelheaded : resolve (E) singlehanded : ambition

The difficulty of this question probably derives primarily from the complexity of the relationship between *headlong* and *forethought* rather than from any inherent difficulty in the words. Analysis of the relationship between *headlong* and *forethought* reveals the following: an action or behavior that is *headlong* is one that lacks *forethought.* Only answer choice (A) displays the same relationship between its two terms.

ANTONYMS

Although antonym questions test knowledge of vocabulary more directly than do any of the other verbal question types, the purpose of the antonym questions is to measure not merely the strength of your vocabulary but also the ability to reason from a given concept to its opposite. Antonyms may require only rather general knowledge of a word, or they may require you to make fine distinctions among answer choices. Antonyms are generally confined to nouns, verbs, and adjectives; answer choices may be single words or phrases.

Some approaches that may be helpful in answering antonym questions:

- Remember that you are looking for the word that is the most nearly *opposite* to the given word; you are *not* looking for a synonym. Since many words do not have a precise opposite, you must look for the answer choice that expresses a concept *most nearly* opposite to that of the given word.
- In some cases more than one of the answer choices may appear at first to be opposite to the given word. Questions that require you to make fine distinctions among two or more answer choices are best handled by defining more precisely or in greater detail the meaning of the given word.
- It is often useful, in weighing answer choices, to make up a sentence using the given word or words. Substituting the answer choices in the phrase or sentence and seeing which best "fits," in that it reverses the meaning or tone of the sentence or phrase, may help you determine the best answer.
- Remember that a particular word may have more than one meaning.
- Use your knowledge of root, prefix, and suffix meanings to help you determine the meanings of words with which you are not entirely familiar.

Directions: **Each question below consists of a word printed in capital letters followed by five lettered words or phrases. Choose the lettered word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, be sure to consider all the choices before deciding which one is best.**

3. DIFFUSE : (A) contend (B) concentrate (C) imply (D) pretend (E) rebel

The best answer is (B). *Diffuse* means to permit or cause to spread out; only (B) presents an idea that is in any way opposite to *diffuse.*

4. MULTIFARIOUS : (A) deprived of freedom (B) deprived of comfort (C) lacking space (D) lacking stability (E) lacking diversity

Multifarious means having or occurring in great variety, so the best answer is (E). Even if you are not entirely familiar with the meaning of *multifarious,* it is possible to use the clue provided by "multi-" to help find the right answer to this question

SENTENCE COMPLETIONS

The purpose of the sentence completion questions is to measure the ability to use the various kinds of cues provided by syntax and grammar to recognize the overall meaning of a sentence. In deciding which of five words or sets of words can best be substituted for blank spaces in a sentence, you must analyze the relationships among the component parts of the incomplete sentence. You must consider each answer choice and decide which completes the sentence in such a way that the sentence has a logically satisfying meaning and can be read as a stylistically integrated whole.

Sentence completion questions provide a context within which to analyze the function of words as they relate to and combine with one another to form a meaningful unit of discourse.

Some approaches that may be helpful in answering sentence completion questions:

- Read the entire incomplete sentence carefully before you consider the answer choices. Be sure you understand the ideas expressed and examine the sentence for possible indications of tone (irony, humor, and the like).
- Before reading the answer choices, you may find it helpful to fill in the blanks with a word or words of your own that complete the meaning of the sentence. Then examine the answer choices to see if any of them parallels your own completion of the sentence.
- Pay attention to grammatical clues in the sentence. For example, words like *although* and *nevertheless* indicate that some qualification or opposition is taking place in the sentence, whereas *moreover* implies an intensification or support of some idea in the sentence.
- If a sentence has two blanks, be sure that *both* parts of your answer choice fit logically and stylistically into the sentence.
- When you have chosen an answer, read the complete sentence through to check that it has acquired a logically and stylistically satisfying meaning.

Directions: **Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words for each blank that *best* fits the meaning of the sentence as a whole.**

5. Early ------- of hearing loss is ------- by the fact that the other senses are able to compensate for moderate amounts of loss, so that people frequently do not know that their hearing is imperfect.
(A) discovery . . indicated
(B) development . . prevented
(C) detection . . complicated
(D) treatment . . facilitated
(E) incidence . . corrected

The statement that the other senses compensate for partial loss of hearing indicates that the hearing loss is not *prevented* or *corrected*; therefore, choices (B) and (E) can be eliminated. Furthermore, the ability to compensate for hearing loss certainly does not facilitate the early *treatment* (D) or the early *discovery* (A) of hearing loss. It is reasonable, however, that early *detection* of hearing loss is *complicated* by the ability to compensate for it. The best answer is (C).

6. The ------- science of seismology has grown just enough so that the first overly bold theories have been -------.
(A) magnetic . . accepted
(B) fledgling . . refuted
(C) tentative . . analyzed
(D) predictive . . protected
(E) exploratory . . recalled

At first reading, there may appear to be more than one answer choice that "makes sense" when substituted in the blanks of the sentence. (A), (C), and (D) can be dismissed fairly readily when it is seen that *accepted, tentative,* and *protected* are not compatible with *overly bold* in the sentence. Of the two remaining choices, (B) is superior on stylistic grounds: theories are not *recalled* (E), and *fledgling* (B) reflects the idea of growth present in the sentence.

READING COMPREHENSION

The purpose of the reading comprehension questions is to measure the ability to read with understanding, insight, and discrimination. This type of question explores your ability to analyze a written passage from several perspectives, including the ability to recognize both explicitly stated elements in the passage and assumptions underlying statements or arguments in the passage as well as the implications of those statements or arguments. Because the written passage upon which reading comprehension questions are based presents a sustained discussion of a particular topic, there is ample context for analyzing a variety of relationships; for example, the function of a word in relation to a larger segment of the passage, the relationships among the various ideas in the passage, or the relation of the author to his or her topic or to the audience.

There are six types of reading comprehension questions. These types focus on (1) the main idea or primary purpose of the passage; (2) information explicitly stated in the passage; (3) information or ideas implied or suggested by the author; (4) possible applications of the author's ideas to other situations, including the identification

of situations or processes analogous to those described in the passage; (5) the author's logic, reasoning, or persuasive techniques; and (6) the tone of the passage or the author's attitude as it is revealed in the language used.

Some reading comprehension questions ask a question like the following: "Which of the following hypothetical situations most closely resembles the situation described in the passage?" Such questions are followed by a series of answer choices that are not explicitly connected to the content of the reading passage but instead present situations or scenarios from other realms, one of which parallels something in the passage in a salient way. You are asked to identify the one answer choice that is most clearly analogous to the situation presented in the passage.

In each edition of the General Test, there are three or more reading comprehension passages, each providing the basis for answering two or more questions. The passages are drawn from different subject matter areas, including the humanities, the social sciences, the biological sciences, and the physical sciences.

Some approaches that may be helpful in answering reading comprehension questions:

- Since reading passages are drawn from many different disciplines and sources, you should not expect to be familiar with the material in all the passages. However, you should not be discouraged by encountering material with which you are not familiar; questions are to be answered on the basis of the information provided in the passage, and you are not expected to rely on outside knowledge, which you may or may not have, of a particular topic.
- Whatever strategy you choose, you should analyze the passage carefully before answering the questions. As with any kind of close and thoughtful reading, you should be sensitive to clues that will help you understand less explicit aspects of the passage. Try to separate main ideas from supporting ideas or evidence; try also to separate the author's own ideas or attitudes from information he or she is simply presenting. It is important to note transitions from one idea to the next and to examine the relationships among the different ideas or parts of the passage. For example, are they contrasting? Are they complementary? You should consider both the points the author makes and the conclusions he or she draws and also how and why those points are made or conclusions drawn.
- Read each question carefully and be certain that you understand exactly what is being asked.
- *Always* read all the answer choices before selecting the best answer.
- The best answer is the one that most accurately and most completely answers the question being posed. Be careful not to pick an answer choice simply because it is a true statement; be careful also not to be misled by answer choices that are only partially true or only partially satisfy the problem posed in the question.
- Answer the questions on the basis of the information provided in the passage and do not rely on outside knowledge. Your own views or opinions may sometimes conflict with the views expressed or the information provided in the passage; be sure that you work within the context provided by the passage. You should not expect to agree with everything you encounter in reading passages.

Directions: **The passage is followed by questions based on its content. After reading the passage, choose the best answer to each question. Answer all questions following the passage on the basis of what is *stated* or *implied* in the passage.**

Picture-taking is a technique both for annexing the objective world and for expressing the singular self. Photographs depict objective realities that already exist, though only the camera can disclose them. And they depict an individual photographer's temperament, discovering itself through the camera's cropping of reality. That is, photography has two antithetical ideals: in the first, photography is about the world, and the photographer is a mere observer who counts for little; but in the second, photography is the instrument of intrepid, questing subjectivity and the photographer is all.

These conflicting ideals arise from a fundamental uneasiness on the part of both photographers and viewers of photographs toward the aggressive component in "taking" a picture. Accordingly, the ideal of a photographer as observer is attractive because it implicitly denies that picture-taking is an aggressive act. The issue, of course, is not so clear-cut. What photographers do cannot be characterized as simply predatory or as simply, and essentially, benevolent. As a consequence, one ideal of picture-taking or the other is always being rediscovered and championed.

An important result of the coexistence of these two ideals is a recurrent ambivalence toward photography's means. Whatever the claims that photography might make to be a form of personal expression on a par with painting, its originality is inextricably linked to the powers of a machine. The steady growth of these powers has made possible the extraordinary informativeness and imaginative formal beauty of many photographs, like Harold Edgerton's high-speed photographs of a bullet hitting its target or of the swirls and eddies of a tennis stroke. But as cameras become more sophisticated, more automated, some photographers are tempted to disarm themselves or to suggest that they are not really armed, preferring to submit themselves to the limits imposed by premodern camera technology because a cruder, less high-powered machine is thought to give more interesting or emotive results, to leave more room for creative accident. For example, it has been virtually a point of honor for many photographers, including Walker Evans and Cartier-Bresson, to refuse to use modern equipment. These photographers have come to doubt the value of the camera as an instrument of "fast seeing." Cartier-Bresson, in fact, claims that the modern camera may see too fast.

This ambivalence toward photographic means determines trends in taste. The cult of the future (of faster and faster seeing) alternates over time with the wish to return to a purer past — when images had a handmade quality. This nostalgia for some pristine state of the photographic enterprise is currently widespread and underlies the present-day enthusiasm for daguerreotypes and the work of forgotten nineteenth-century provincial photographers. Photographers and viewers of photographs, it seems, need periodically to resist their own knowingness.

7. According to the passage, the two antithetical ideals of photography differ primarily in the

(A) value that each places on the beauty of the finished product

(B) emphasis that each places on the emotional impact of the finished product

(C) degree of technical knowledge that each requires of the photographer

(D) extent of the power that each requires of the photographer's equipment

(E) way in which each defines the role of the photographer

The best answer to this question is (E). Photography's two ideals are presented in lines 7-11. The main emphasis in the description of these two ideals is on the relationship of the photographer to the enterprise of photography, with the photographer described in the one as a passive observer and in the other as an active questioner. (E) identifies this key feature in the description of the two ideals — the way in which each ideal conceives or defines the role of the photographer in photography. (A) through (D) present aspects of photography that are mentioned in the passage, but none of these choices represents a primary difference between the two ideals of photography.

8. According to the passage, interest among photographers in each of photography's two ideals can best be described as

(A) rapidly changing

(B) cyclically recurring

(C) steadily growing

(D) unimportant to the viewers of photographs

(E) unrelated to changes in technology

This question requires one to look for comments in the passage about the nature of photographers' interest in the two ideals of photography. While the whole passage is, in a sense, about the response of photographers to these ideals, there are elements in the passage that comment specifically on this issue. Lines 20-22 tell us that the two ideals alternate in terms of their perceived relevance and value, that each ideal has periods of popularity and of neglect. These lines support (B). Lines 23-25 tell us that the two ideals affect attitudes toward "photography's means," that is, the technology of the camera; (E), therefore, cannot be the best answer. In lines 46-49, attitudes toward photographic means (which result from the two ideals) are said to alternate over time; these lines provide further support for (B). (A) can be eliminated because, although the passage tells us that the interest of photographers in each of the ideals fluctuates over time, it nowhere indicates that this fluctuation or change is rapid. Nor does the passage say anywhere that interest in these ideals is growing; the passage *does* state that the powers of the camera are steadily growing (line 28), but this does not mean that interest in the two ideals is growing. Thus (C) can be eliminated. (D) can be eliminated because the passage nowhere states that reactions to the ideals are either important or unimportant to viewers' concerns. Thus (B) is the best answer.

QUANTITATIVE ABILITY

The quantitative section of the General Test is designed to measure basic mathematical skills, and understanding of elementary mathematical concepts, as well as the ability to reason quantitatively and to solve problems in a quantitative setting.

In general, the mathematics required does not extend beyond that usually covered in high school. It is expected that examinees are familiar with conventional symbolism, such as $x < y$ (x is less than y) and $x \neq y$ (x is not equal to y), $m \parallel n$ (line m is parallel to line n), $m \perp n$ (line m is perpendicular to line n), and the symbol for a right angle in a figure: 

($\angle ABC$ is a right angle).

Also, standard mathematical conventions are used in the test questions unless otherwise indicated. For example, numbers are in base 10, the positive direction of a number line is to the right, and distances are nonnegative. Whenever nonstandard notation or conventions are used in a question, they are explicitly introduced in the question.

Many of the questions are posed as word problems in a real-life setting, with quantitative information given in the text of a question or in a table or graph of data. Other questions are posed in a pure-math setting that may include a geometric figure, a graph, or a coordinate system. The following conventions about numbers and figures are used in the quantitative section.

Numbers and Units of Measurement

All numbers used are real numbers.

Numbers are to be used as exact numbers, even though in some contexts they are likely to have been rounded. For example, if a question states that "30 percent of the company's profit was from health products," then 30% is to be used as an exact percent; it is not to be used as a rounded number obtained from, say, 29% or 30.1%.

An integer that is given as the number of objects in a real-life or pure-math setting is to be taken as the total number of these objects. For example, if a question states that "a bag contains 50 marbles, and 23 of the marbles are red," then 50 is to be taken as the total number of marbles in the bag and 23 is to be taken as the total number of red marbles in the bag, so that the other 27 marbles are <u>not</u> red.

Questions may involve units of measurement such as English units or metric units. If an answer to a question requires converting one unit of measurement to another, then the relationship between the units is provided, unless the relationship is a common one, such as minutes to hours, or centimeters to meters.

Figures

Geometric figures that accompany questions provide information useful in answering the questions. However, unless a note states that a geometric figure is drawn to scale, you should solve these problems not by estimating sizes by sight or by measurement, but by reasoning about geometry.

Geometric figures consist of points, lines (or line segments), curves (such as circles), angles, regions, etc., and labels that identify these objects or their sizes. (Note that geometric figures may appear somewhat jagged on a computer screen.)

Geometric figures are assumed to lie in a plane unless otherwise indicated.

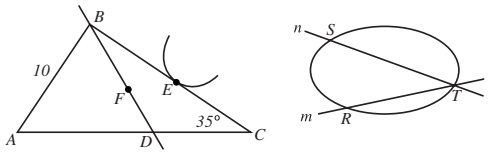
Points are indicated by a dot, a label, or the intersection of two or more lines or curves.

Points on a line or curve are assumed to be in the order shown; points that are on opposite sides of a line or curve are assumed to be oriented as shown.

Lines shown as straight are assumed to be straight (though they may look jagged on a computer screen). When curves are shown, they are assumed to be not straight.

Angle measures are assumed to be positive and less than or equal to 360 degrees.

To illustrate some of these conventions, consider the following geometric figures.



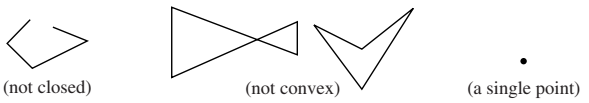
In the figures, it can be determined that

- *ABD* and *DBC* are triangles.
- Points *A, D,* and *C* lie on a straight line, so *ABC* is also a triangle.
- Point *D* is a distinct point between points *A* and *C*.
- Point *E* is the only intersection point of line segment *BC* and the small curve shown.
- Points *A* and *E* are on opposite sides of line *BD*.
- Point *F* is on line segment *BD*.
- The length of line segment *AD* is less than the length of line segment *AC*.
- The length of line segment *AB* is 10.
- The measure of angle *ABD* is less than the measure of angle *ABC*.
- The measure of angle *ACB* is 35 degrees.
- Lines *m* and *n* intersect the closed curve at three points: *R, S,* and *T*.

From the figures, it cannot be determined whether

- The length of line segment *AD* is greater than the length of line segment *DC*.
- The measures of angles *BAD* and *BDA* are equal.
- The measure of angle *ABD* is greater than the measure of angle *DBC*.
- Angle *ABC* is a right angle.

When a square, circle, polygon, or other closed geometric figure is described in words with no picture, the figure is assumed to enclose a convex region. It is also assumed that such a closed geometric figure is not just a single point. For example, a quadrilateral cannot be any of the following:



When graphs of real-life data accompany questions, they are drawn as accurately as possible so you can read or estimate data values from the graphs (whether or not there is a note that the graphs are drawn to scale).

Standard conventions apply to graphs of data unless otherwise indicated. For example, a circle graph represents 100 percent of the data indicated in the graph's title, and the areas of the individual sectors are proportional to the percents they represent. Scales, gridlines, dots, bars, shadings, solid and dashed lines, legends, etc., are used on graphs to indicate the data. Sometimes, scales that do not begin at zero are used, as well as broken scales.

Coordinate systems such as number lines and *xy*-planes are generally drawn to scale.

ARITHMETIC

Questions that test *arithmetic* include those involving the following topics: arithmetic operations (addition, subtraction, multiplication, division, and powers) on real numbers, operations on radical expressions, the number line, estimation, percent, absolute value, properties of integers (for example, divisibility, factoring, prime numbers, and odd and even integers).

Some facts about arithmetic that may be helpful

For any two numbers on the number line, the number on the left is less than the number on the right; for example, -4 is to the left of -3, which is to the left of 0.

The sum and product of signed numbers will be positive or negative depending on the operation and the signs of the numbers; for example, the product of a negative number and a positive number is negative.

Division by zero is undefined; that is, $\frac{x}{0}$ is not a real number for any x.

If n is a positive integer, then x^n denotes the product of n factors of x; for example, 3^4 means $(3)(3)(3)(3) = 81$. If $x \neq 0$, then $x^0 = 1$.

Squaring a number between 0 and 1 (or raising it to a higher power) results in a smaller number; for example, $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ and $(0.5)^3 = 0.125$.

An odd integer power of a negative number is negative, and an even integer power is positive; for example, $(-2)^3 = -8$ and $(-2)^2 = 4$.

The radical sign $\sqrt{\ }$ means "the nonnegative square root of;" for example, $\sqrt{0} = 0$ and $\sqrt{4} = 2$. The negative square root of 4 is denoted by $-\sqrt{4} = -2$. If $x < 0$, then $\sqrt{x}$ is not a real number; for example, $\sqrt{-4}$ is not a real number.

The *absolute value* of x, denoted by $|x|$, is equal to x if $x \geq 0$ and equal to $-x$ if $x < 0$; for example, $|8| = 8$ and $|-8| = -(-8) = 8$.

If n is a positive integer, then $n!$ denotes the product of all positive integers less than or equal to n; for example, $4! = (4)(3)(2)(1) = 24$.

The sum and product of even and odd integers will be even or odd depending on the operation and the kinds of integers; for example, the sum of an odd integer and an even integer is odd.

If an integer P is a divisor (also called a factor) of another integer N, then N is the product of P and another integer, and N is said to be a multiple of P; for example, 3 is a divisor, or a factor, of 6, and 6 is a multiple of 3.

A *prime* number is a positive integer that has only two distinct positive divisors: 1 and itself. For example, 2, 3, 5, 7, and 11 are prime numbers, but 9 is not a prime number because it has three positive divisors: 1, 3, and 9.

ALGEBRA
(including coordinate geometry)

Questions that test *algebra* include those involving the following topics: rules of exponents, factoring and simplifying algebraic expressions, concepts of relations and functions, equations and inequalities, and coordinate geometry (including slope, intercepts, and graphs of equations and inequalities). The skills required include the ability to solve linear and quadratic equations and inequalities, and simultaneous equations; the ability to read a word problem and set up the necessary equations or inequalities to solve it; and the ability to apply basic algebraic skills to solve problems.

Some facts about algebra that may be helpful

If $ab = 0$, then $a = 0$ or $b = 0$; for example, if $(x - 1)(x + 2) = 0$, it follows that either $x - 1 = 0$ or $x + 2 = 0$; therefore, $x = 1$ or $x = -2$.

Adding a number to or subtracting a number from both sides of an equation preserves the equality. Similarly, multiplying or dividing both sides of an equation by a nonzero number preserves the equality. Similar rules apply to inequalities, except that multiplying or dividing both sides of an inequality by a *negative* number reverses the inequality. For example, multiplying the inequality $3x - 4 > 5$ by 4 yields the inequality $12x - 16 > 20$; however, multiplying that same inequality by -4 yields $-12x + 16 < -20$.

The following rules for exponents may be useful. If *r, s, x,* and *y* are positive numbers, then

(a) $x^{-r} = \frac{1}{x^r}$; for example, $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

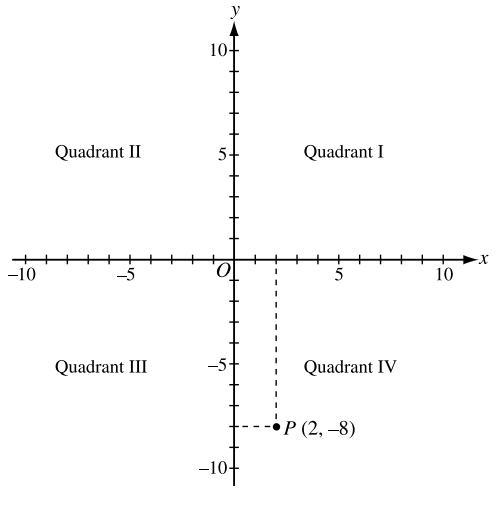
(b) $(x^r)(x^s) = x^{r+s}$; for example, $(3^2)(3^4) = 3^6 = 729$

(c) $(x^r)(y^r) = (xy)^r$; for example, $(3^4)(2^4) = 6^4 = 1{,}296$

(d) $(x^r)^s = x^{rs}$; for example, $(2^3)^4 = 2^{12} = 4{,}096$

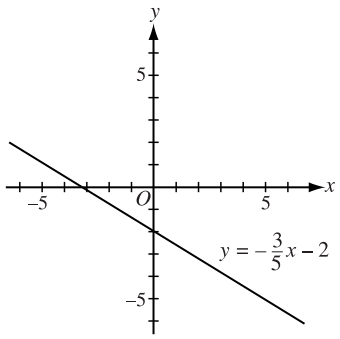
(e) $\frac{x^r}{x^s} = x^{r-s}$; for example, $\frac{4^2}{4^5} = 4^{2-5} = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

The rectangular coordinate plane, or *xy*-plane, is shown below.



The *x*-axis and *y*-axis intersect at the origin *O*, and they partition the plane into four quadrants, as shown. Each point in the plane has coordinates (*x, y*) that give its location with respect to the axes; for example, the point $P(2, -8)$ is located 2 units to the right of the *y*-axis and 8 units below the *x*-axis. The units on the *x*-axis are the same length as the units on the *y*-axis, unless otherwise noted.

Equations involving the variables *x* and *y* can be graphed in the *xy*-plane. For example, the graph of the linear equation $y = -\frac{3}{5}x - 2$ is a line with a slope of $-\frac{3}{5}$ and a *y*-intercept of –2, as shown below.

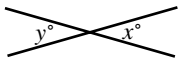


GEOMETRY

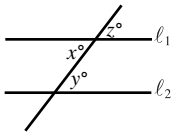
Questions that test *geometry* include those involving the following topics: properties associated with parallel lines, circles, triangles (including isosceles, equilateral, and $30^\circ - 60^\circ - 90^\circ$ triangles), rectangles, other polygons, area, perimeter, volume, the Pythagorean Theorem, and angle measure in degrees. The ability to construct proofs is not measured.

Some facts about geometry that may be helpful

If two lines intersect, then the opposite angles (called vertical angles) are equal; for example, in the figure below, $x = y$.



If two parallel lines are intersected by a third line, certain angles that are formed are equal. As shown in the figure below, if $\ell_1 \parallel \ell_2$, then $x = y = z$.

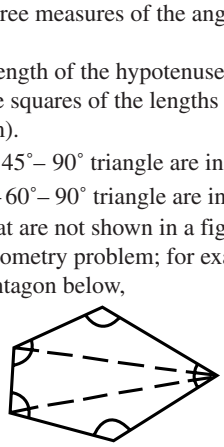


The sum of the degree measures of the angles of a triangle is 180.

The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the two legs (Pythagorean Theorem).

The sides of a $45^\circ - 45^\circ - 90^\circ$ triangle are in the ratio $1 : 1 : \sqrt{2}$, and the sides of a $30^\circ - 60^\circ - 90^\circ$ triangle are in the ratio $1 : \sqrt{3} : 2$.

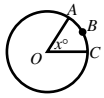
Drawing in lines that are not shown in a figure can sometimes be helpful in solving a geometry problem; for example, by drawing the dashed lines in the pentagon below,



the total number of degrees in the angles of the pentagon can be found by adding the number of degrees in each of the three triangles: 180 + 180 + 180 = 540.

The number of degrees of arc in a circle is 360.

If O is the center of the circle in the figure below, then the length of arc ABC is $\frac{x}{360}$ times the circumference of the circle.



The volume of a rectangular solid or a right circular cylinder is the product of the area of the base and the height; for example, the volume of a cylinder with a base of radius 2 and a height of 5 is $\pi(2^2)(5) = 20\pi$.

DATA ANALYSIS

Questions that test *data analysis* include those involving the following topics: basic descriptive statistics (such as mean, median, mode, range, standard deviation, and percentiles), interpretation of data given in graphs and tables (such as bar and circle graphs, and frequency distributions), and elementary probability. The questions assess the ability to synthesize information, to select appropriate data for answering a question, and to determine whether or not the data provided are sufficient to answer a given question. The emphasis in these questions is on the understanding of basic principles (for example, basic properties of normal distribution) and reasoning within the context of given information.

Some facts about descriptive statistics and probability that may be helpful

In a distribution of n measurements, the (arithmetic) *mean* is the sum of the measurements divided by n. The *median* is the middle measurement after the measurements are ordered by size if n is odd, or it is the mean of the two middle measurements if n is even. The mode is the most frequently occurring measurement (there may be more than one mode). The *range* is the difference between the greatest measurement and the least measurement. Thus, for the measurements: 70, 72, 72, 76, 78, and 82, the mean is $450 \div 6 = 75$, the median is $(72 + 76) \div 2 = 74$, the mode is 72, and the range is 12.

The probability that an event will occur is a value between 0 and 1, inclusive. If p is the probability that a particular event will occur, then $0 \le p \le 1$, and the probability that the event will *not* occur is $1 - p$. For example, if the probability is 0.85 that it will rain tomorrow, then the probability that it will not rain tomorrow is $1 - 0.85 = 0.15$.

The quantitative measure employs two types of questions: quantitative comparison and problem solving.

QUANTITATIVE COMPARISON

The quantitative comparison questions test the ability to reason quickly and accurately about the relative sizes of two quantities or to perceive that not enough information is provided to make such a comparison. To solve a quantitative comparison problem, you must compare the quantities that are given in two columns, Column A and Column B, and decide whether one quantity is greater than the other, whether the two quantities are equal, or whether the relationship cannot be determined from the information given. Information about the two quantities is given in the columns themselves or may be centered above the columns. Here are some examples with the correct answers indicated according to the following answer choices.

(A) The quantity in Column A is greater.
(B) The quantity in Column B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

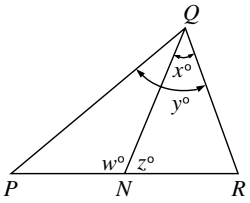
	Column A	Column B	Correct Answer
Example 1:	2^3	3^2	(A) ● (C) (D) (E)
Example 2:	**The smallest prime number greater than 20**	**23**	(A) (B) ● (D) (E)
	***m* is an integer.**		
Example 3:	$3m + 7$	**7**	(A) (B) (C) ● (E) **(since *m* can be positive, negative, or zero)**

Some questions only require some manipulation to determine which of the quantities is greater; other questions require more reasoning or thinking of special cases in which the relative sizes of the quantities are reversed.

The following strategies may help in answering quantitative comparison questions.

- Do not waste time performing needless computations in order to eventually compare two specific numbers. Simplify or transform one or both of the given quantities only as much as is necessary to determine which quantity is greater or whether the two quantities are equal. If you determine that one quantity is greater than the other, do not take time to find the exact sizes of the quantities. Answer and go on to the next question.
- Consider all kinds of appropriate real numbers before you make a decision. As soon as you establish that the quantity in one column is greater in one case while the quantity in the other column is greater in another case, choose "The relationship cannot be determined from the information given" and move on to the next question.
- Geometric figures may not be drawn to scale. Comparisons should be based on the given information together with your knowledge of mathematics rather than on the exact appearance of the figure. You can sometimes find a clue by sketching another figure that conforms to the information given. (Scratch paper will be provided.) Try to visualize the parts of the figure that are fixed by the information given and the parts that are changeable. If the figure can be changed in such a way that the relative sizes of the quantities in the columns are reversed while still conforming to the information given, then the answer is "The relationship cannot be determined from the information given."

Here are some more examples:

	Column A	Column B	Correct Answer
Examples 4-6 refer to ΔPQR.			
			
Example 4:	***PN***	***NR***	Ⓐ Ⓑ Ⓒ ● Ⓔ **(since equal measures cannot be assumed, even though *PN* and *NR* appear to be equal)**
Example 5:	***x***	***y***	Ⓐ ● Ⓒ Ⓓ Ⓔ **(since *N* is between *P* and *R*)**
Example 6:	$w + z$	**180**	Ⓐ Ⓑ ● Ⓓ Ⓔ **(since *PR* is a straight line)**
	A machine was in operation for *t* minutes.		
Example 7:	**The number of seconds that the machine was in operation**	$60t$	Ⓐ Ⓑ ● Ⓓ Ⓔ
	A farmer has two plots of land that are equal in area. The first plot is divided into 16 parcels with *m* acres in each parcel, and the second plot is divided into 20 parcels with *n* acres in each parcel.		
Example 8:	***m***	***n***	● Ⓑ Ⓒ Ⓓ Ⓔ

Directions: **Each of the sample questions consists of two quantities, one in Column A and one in Column B. There may be additional information, centered above the two columns, that concerns one or both of the quantities. A symbol that appears in both columns represents the same thing in Column A as it does in Column B.**

You are to compare the quantity in Column A with the quantity in Column B and decide whether:

(A) The quantity in Column A is greater.
(B) The quantity in Column B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Note: Since there are only four choices, NEVER MARK (E).

	Column A	Column B
1.	**9.8**	$\sqrt{100}$

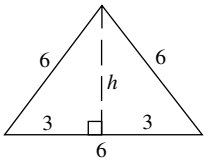
$\sqrt{100}$ denotes 10, the positive square root of 100. (For any positive number x, $\sqrt{x}$ denotes the *positive* number whose square is x.) Since 10 is greater than 9.8, the best answer is (B). It is important not to confuse this question with a comparison of 9.8 and x where $x^2 = 100$. The latter comparison would yield (D) as the correct answer because $x^2 = 100$ implies that either $x = 10$ or $x = -10$, and there would be no way to determine which value x would actually have.

2.	$(-6)^4$	$(-6)^5$

Since $(-6)^4$ is the product of four negative factors, and the product of an even number of negative numbers is positive, $(-6)^4$ is positive. Since the product of an odd number of negative numbers is negative, $(-6)^5$ is negative. Therefore, $(-6)^4$ is greater than $(-6)^5$ since any positive number is greater than any negative number. The best answer is (A). It is not necessary to calculate that $(-6)^4 = 1{,}296$ and that $(-6)^5 = -7{,}776$ in order to make the comparison.

3.	**The area of an equilateral triangle with side 6**	**The area of a right triangle with legs $\sqrt{3}$ and 9**

The area of a triangle is one half the product of the lengths of the base and the altitude. In Column A, the length of the altitude must first be determined. A sketch of the triangle may be helpful.



The altitude h divides the base of an equilateral triangle into two equal parts. From the Pythagorean Theorem, $h^2 + 3^2 = 6^2$, or $h = 3\sqrt{3}$. Therefore, the area of the triangle in Column A is $\left(\frac{1}{2}\right)(6)(3\sqrt{3}) = 9\sqrt{3}$. In Column B, the base and the altitude of the right triangle are the two legs; therefore, the area is $\left(\frac{1}{2}\right)(9)(\sqrt{3}) = \frac{9\sqrt{3}}{2}$. Since $9\sqrt{3}$ is greater than $\frac{9\sqrt{3}}{2}$, the best answer is (A).

$$x^2 = y^2 + 1$$

4.	***x***	***y***

From the given equation, it can be determined that $x^2 > y^2$; however, the relative sizes of x and y cannot be determined. For example, if $y = 0$, then x could be 1 or -1 and, since there is no way to tell which number x is, the best answer is (D).

Column A	Column B

Class	Class Size	Mean Score
1	50	89
2	30	81
3	20	85

5. Three classes took the same psychology test. The class sizes and (arithmetic) mean scores are shown.

The overall (arithmetic) mean score for the 3 classes	**85**

The overall mean score could be found by weighting each mean score by class size and dividing the result by 100, the total of all the class sizes, as follows.

$$\frac{(50)(89)+(30)(81)+(20)(85)}{100} = 85.8$$

Therefore, the best answer is (A). However, the calculations are unnecessary; classes 1 and 2 must have a mean greater than 85 since the mean of 89 and 81 is 85 and there are 20 more students in class 1 than in class 2. Since class 3 has a mean of 85, it must be true that the overall mean for the 3 classes is greater than 85.

PROBLEM SOLVING

The problem solving questions are standard multiple choice questions with five answer choices. To answer a question, select the best of the answer choices. Some problem solving questions are discrete while others occur in sets of two to five questions that share common information. For some of the questions, the solution requires only simple computations or manipulations; for others, the solution requires multi-step problem solving.

The following strategies may be helpful in answering problem solving questions.

- Read each question carefully to determine what information is given and what is being asked.
- Before attempting to answer a question, scan the answer choices; otherwise you may waste time putting answers in a form that is not given (for example, putting an answer in the form $\frac{\sqrt{2}}{2}$ when the answer choice is given in the form $\frac{1}{\sqrt{2}}$, or finding the answer in decimal form, such as 3.25, when the answer choices are given in fractional form, such as $3\frac{1}{4}$).
- For questions that require approximations, scan the answer choices to get some idea of the required closeness of approximation; otherwise you may waste time on long computations when a short mental process would be sufficient (for example, finding 48 percent of a number when taking half of the number would give a close enough approximation).

Directions for problem solving questions and some examples of discrete questions with explanations follow.

Directions: **Each of the following questions has five answer choices. For each of these questions, select the best of the answer choices given.**

6. The average (arithmetic mean) of x and y is 20. If $z = 5$, what is the average of x, y, and z?

(A) $8\frac{1}{3}$ (B) 10 (C) $12\frac{1}{2}$ (D) 15 (E) $17\frac{1}{2}$

Since the average of x and y is 20, $\frac{x+y}{2} = 20$, or $x + y = 40$. Thus $x + y + z = x + y + 5 = 40 + 5 = 45$, and therefore $\frac{x+y+z}{3} = \frac{45}{3} = 15$. The best answer is (D).

7. In a certain year, Minnesota produced $\frac{2}{3}$ and Michigan produced $\frac{1}{6}$ of all the iron ore produced in the United States. If all the other states combined produced 18 million tons that year, how many million tons did Minnesota produce that year?

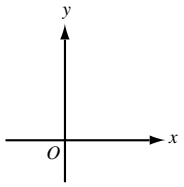
(A) 27 (B) 36 (C) 54 (D) 72 (E) 162

Since Minnesota produced $\frac{2}{3}$ and Michigan produced $\frac{1}{6}$ of all the iron ore produced in the United States, the two states together produced $\frac{5}{6}$ of the iron ore. Therefore, the 18 million tons produced by the rest of the United States was $\frac{1}{6}$ of the total production. Thus the total United States production was (6)(18) = 108 million tons, and Minnesota produced $\frac{2}{3}(108) = 72$ million tons. The best answer is (D).

8. If $\frac{x}{3} - \frac{x}{6} + \frac{x}{9} - \frac{x}{12} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$, then x =

(A) 3 (B) 1 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$ (E) −3

This problem can be solved without a lot of computation by factoring $\frac{x}{3}$ out of the expression on the left side of the equation, i.e., $\frac{x}{3} - \frac{x}{6} + \frac{x}{9} - \frac{x}{12} = \frac{x}{3}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right)$, and substituting the factored expression into the equation, obtaining $\frac{x}{3}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$. Dividing both sides of the equation by $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$ (which is not zero) gives the resulting equation $\frac{x}{3} = 1$. Thus $x = 3$ and the best answer is (A).



9. If the equation $y = 3x - 18$ were graphed on the coordinate axes above, the graph would cross the y-axis at the point (x, y) where

(A) $x = 0$ and $y = 18$
(B) $x = 0$ and $y = -18$
(C) $x = 0$ and $y = 6$
(D) $x = 6$ and $y = 0$
(E) $x = -6$ and $y = 0$

A graph crosses the y-axis at a point (x, y) where $x = 0$. In the given equation, when $x = 0$, $y = 3(0) - 18 = -18$. Therefore, the graph would cross the y-axis at the point $(0, -18)$, and the best answer is (B).

10. The operation denoted by the symbol ♦ is defined for all real numbers p and r as follows.

$$p \blacklozenge r = pr - p + r$$

What is the value of $(-4) \blacklozenge 5$?

(A) -9
(B) -11
(C) -19
(D) 19
(E) 21

By the definition,

$$(-4) \blacklozenge 5 = (-4)(5) - (-4) + 5 = -20 + 4 + 5 = -11,$$

and therefore the best answer is (B).

Some problem solving questions involve data analysis; many of these occur in sets of two to five questions that share common data in the form of tables, graphs, etc. In questions that involve data analysis, graphs are drawn as accurately as possible. Therefore, you can read or estimate data values from the graphs (whether or not there is a note that the graphs are drawn to scale).

The following strategies may help in answering problem solving questions that involve data analysis.

- Scan the data briefly to see what it is about, but do not attempt to analyze it in too much detail before reading the questions. Focus on those aspects of the data that are necessary to answer the questions. Be sure to read all notes related to the data.
- When possible, try to make visual comparisons of the data given in a graph and estimate products and quotients rather than perform involved computations.
- Remember that these questions are to be answered only on the basis of the data given, everyday facts (such as the number of days in a year), and your knowledge of mathematics. Do not make use of specific information you recall that may seem to relate to the particular situation on which the questions are based unless that information can be derived from the data provided.

Some examples of problem solving questions involving data analysis, with explanations, follow.

Questions 11-13 refer to the following table.

PERCENT CHANGE IN DOLLAR AMOUNT OF SALES IN CERTAIN RETAIL STORES FROM 1977 TO 1979

Store	Percent Change From 1977 to 1978	Percent Change From 1978 to 1979
P	+10	−10
Q	−20	+9
R	+5	+12
S	−7	−15
T	+17	−8

11. In 1979, for which of the stores was the dollar amount of sales greater than that of any of the others shown?
(A) *P* (B) *Q* (C) *R* (D) *S*
(E) It cannot be determined from the information given.

Since the only information given in the table is the percent change from year to year, there is no way to compare the dollar amount of sales for the stores in 1979 or in any other year. The best answer is (E).

12. In store *T*, the dollar amount of sales for 1978 was approximately what percent of the dollar amount of sales for 1979?
(A) 86% (B) 92% (C) 109% (D) 117% (E) 122%

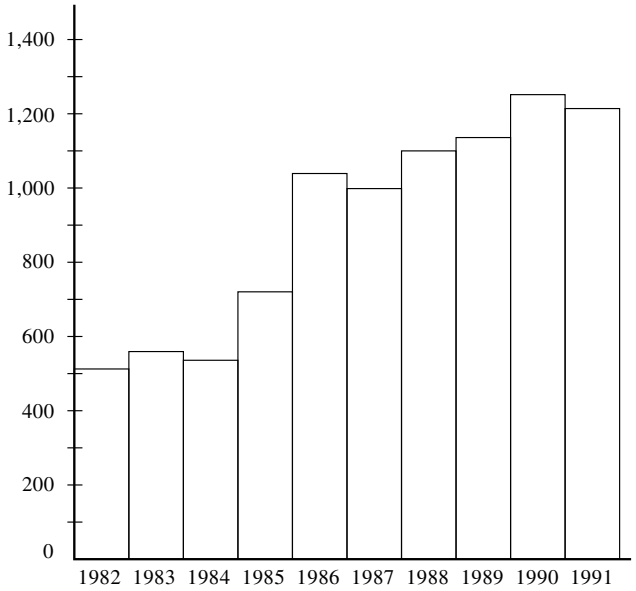
If A is the amount of sales for store T in 1978, then $0.08A$ is the amount of decrease and $A - 0.08A = 0.92A$ is the amount of sales for 1979. Therefore, the desired result can be obtained by dividing A by $0.92A$, which equals $\frac{1}{0.92}$, or approximately 109%. The best answer is (C).

13. If the dollar amount of sales in store *P* was $800,000 in 1977, what was the dollar amount of sales in that store in 1979?
(A) $727,200 (B) $792,000 (C) $800,000
(D) $880,000 (E) $968,000

If sales in store P were $800,000 in 1977, then in 1978 they were 110 percent of that, i.e., $880,000. In 1979 sales were 90 percent of $880,000, i.e., $792,000. Note that an increase of 10 percent in one year and a decrease of 10 percent in the following year does not result in the same dollar amount as the original dollar amount of sales because the base used in computing the percents changes from $800,000 to $880,000. The best answer is (B).

Questions 14-15 refer to the following graph.

NUMBER OF GRADUATE STUDENT APPLICANTS AT UNIVERSITY *X*, 1982-1991



14. In which of the following years did the number of graduate student applicants increase the most from that of the previous year?

(A) 1985
(B) 1986
(C) 1988
(D) 1990
(E) 1991

This question can be answered directly by visually comparing the heights of the bars in the graph. The greatest increase in height between two adjacent bars occurs for the years 1985 and 1986. The best answer is (B).

15. Which of the following statements can be inferred from the graph?

I. The number of graduate student applicants more than doubled from 1982 to 1991.
II. For each of the years 1983 to 1991, inclusive, the number of graduate student applicants was greater than that of the previous year.
III. The greatest number of graduate students attended University *X* in 1990.

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

For this type of question it is helpful to consider each statement separately. Statement I is true because, as shown in the graph, the number of applicants in 1982 was below 600 and the number in 1991 was above 1,200. Statement II is false because there are three years in which the number of applicants decreased from that of the previous year, namely 1984, 1987, and 1991. Statement III cannot be inferred from the graph because the graph shows only the number of applicants and gives no information about the number of students attending University *X*. Therefore, statement I only can be inferred from the graph, and the best answer is (A).